SYMMETRY: key to nature and natural philosophy

By Eliseo Fernández

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Scope, origins, and organization

This outstanding collection of articles on the role of symmetry in physical theories originated in a workshop held at Oxford in January 2001. Most of the included contributions were first presented there or later composed by the workshop’s participants. The authors represent an impressive array of prominent names in the philosophy of physics at the start of the 21st century.

Katherine Brading and Elena Castellani – in addition to selecting, integrating and annotating the whole – have authored individual contributions and collaborated on the Introduction. Their initial article provides a brief background review of the notions of symmetry and invariance and the history of their slow emergence from obscure beginnings in ancient Greek speculation to their present dominant role in contemporary physics. Much of this material was subsequently incorporated into an expanded essay, “Symmetry and Symmetry Breaking,” the editors’ contribution to *The Stanford Encyclopedia of Philosophy*. It is available online at <http://plato.stanford.edu/archives/fall2003/entries/symmetry-breaking/>. The introduction also serves to articulate the collection’s overall plan, summarizing the topics to be treated in each separate section.

The overall conception of the book resembles that of a similar collection, *Interpreting Bodies*, edited by Castellani and published by Princeton University Press in 1998. There the attention turned to the entangled notions of matter, body, and individuality — issues that partially overlap and complement the subjects addressed by the work under review.
The goals of the Brading and Castellani collection are clearly stated and skillfully fulfilled through the uniform high quality of the contributions, the wise and thoughtful selection of topics, and the appealing device of preceding the four separate parts with well-chosen fragments of classical texts, taken from seminal contributions to the ideas examined. The inclusion of this material is inspirational and motivates the subsequent discussion in a most attractive and efficient manner.

**Why bother about symmetry?**

In spite of the efforts of the contributors to make them accessible to a wide audience, several of the papers included are partially beyond the grasp of those philosophers lacking considerable training in theoretical physics and previous acquaintance with the abstract nature of present discussions in the philosophy of physics. It is to be hoped that this aspect of the book will not discourage philosophers with lesser expertise from reading the more accessible parts. The excellent bibliographies appended to the articles provide sound references to the required background materials, and in most cases it is possible to skip some technical arguments without losing grasp of their main import. It would be a loss indeed for philosophers to be unaware of this vast tangle of arguments and problems. Perhaps one of the most important realizations these readings afford us is that of the considerable weight the outcomes of these issues may bear on some central and perennial questions of general metaphysics and epistemology.

Before considering the individual contributions I will start by briefly sketching some of the basic ideas under consideration. If this somewhat sketchy *aperçu* motivates the potential reader to become engaged with the book it will fulfill its mission, since the much deeper and detailed treatment offered there will quickly remedy its deficiencies.
Groups, symmetries, and invariances

Notions akin to those of symmetry and invariance can be discerned with hindsight in the arguments of physicists dating at least from the times of Galileo and Huygens. The intuitive notion of symmetry refers to the possibility of performing an operation or transformation upon a thing or situation in such a way that some well-defined aspect of it is left unchanged (invariant).

An explicit and precise conception of this idea had to wait until the mid-19th century when a key mathematical structure was recognized and applied in algebra. Specifically, this is the concept of group.

A group $G$ is a set of elements $e$ with an abstract structure determined by the properties of a composition operation (usually called product and denoted “.”) that combines pairs of elements into new elements. These defining properties are:

1. The product is associative: $e_1 \cdot (e_2 \cdot e_3) = (e_1 \cdot e_2) \cdot e_3$
2. There is in $G$ a special (neutral) element $e_n$, such that for every $e$ of $G$ we have $e \cdot e_n = e_n \cdot e = e$
3. For every $e$ of $G$ there is an inverse element $e_i$ such $e \cdot e_i = e_i \cdot e = e_n$.

For example, the set of all real numbers except zero is a group with respect to the operation of multiplication, with $e_n = 1$ and $e_i = 1/e$. When the operation is non-commutative ($e_1 \cdot e_2 \neq e_2 \cdot e_1$) we have what is called a “non-abelian” group.

The idea of group made possible a clear and precise definition of symmetry as invariance under a group of transformations.

By interpreting the elements of a group as representing various relevant transformations, it was possible to find very fruitful applications of these notions, first in algebra (solutions of equations, Galois theory) and then in geometry. Geometrical properties that remained invariant under various groups of
transformations became the basis of Klein’s famous Erlangen Program. This program led to an important classification and unification of geometric theories as a hierarchy of increasing levels of generality, reflecting relations of various groups of transformations to their subgroups.

In the physical sciences, group-theoretic considerations found their first successful applications in crystallography, in the classification of symmetric arrangements of atoms. But the unparalleled role of symmetry in physics in the 20th century arose from a momentous epistemological move first fully realized in the special theory of relativity: that of shifting attention from the invariances found in the phenomena to the symmetries displayed by the laws of nature themselves.

**Types of symmetries**
The entire history of major developments of 20th century physics (in relativity, quantum theory, and condensed matter physics) shows the progressive unfolding of discoveries and applications of new kinds of symmetries, together with their relations to the associated phenomenon of symmetry breaking. Symmetries can be sorted into discrete and continuous. Examples of discrete symmetries are \( P \) (parity – mirror image invariance), \( C \) (charge conjugation invariance), \( T \) (time inversion invariance), and permutation invariance. Examples of continuous symmetries are space translations and rotations. These are invariances with respect to continuous (Lie) groups of transformations.

Another important classification, contributed by Wigner, is that of **geometrical** versus **dynamical** symmetries. Geometrical symmetries refer to invariances of the laws of nature under general transformations in space and time. Examples of geometrical symmetries are the invariances of the laws of nature under translations in space and in time. They entail the conservation laws of momentum and energy, respectively, according to Noether’s first theorem. Dynamical symmetries, on the other hand, refer to invariance in the form of the laws governing individual kinds of interactions (e.g., gravitational, electrodynamical, etc.) with respect to transformations that are local (i.e., not
related to global properties of spacetime). The most important kind of dynamical or local symmetries in contemporary physics is that of “gauge” or phase invariance. Local, place-to-place, variations in a physical quantity are compensated (left invariant) by the introduction of suitably designed gauge fields. The group of transformations defining the symmetry suggests the mathematical form and, at times, the very existence of these fields. Weyl first developed this idea in 1918, in connection with a failed attempt to unify the relativistic theory of gravity with electromagnetism. Later on he applied it successfully to quantum physics. Here, the invariance no longer refers to global spacetime symmetries. It issues, for example, through the possibility of multiplying the complex phase of an electron’s wave function by arbitrary functions without changing the corresponding outcomes of the computation. Yang and Mills rediscovered Weyl’s ideas in the early fifties. They and other physicists applied them, under the generalized form of non-abelian field theories, to the study of the nuclear forces. Today physicists regard all four fundamental interactions of nature (i.e., gravity, electromagnetism, and the weak and strong nuclear forces) as the effects of gauge fields. At present our entire conception of the physical world — from elementary particles to the entire universe — is based on the Standard Model of particle physics. This grand scheme provides, in terms of these gauge invariances, a unified account of the fundamental forces (excepting gravity) and of all the particles of nature.

Broken symmetries
Symmetry breaking is best illustrated by some everyday phenomena, such as water undergoing a “phase transition,” changing from the liquid to the solid state in a kitchen freezer. In the liquid state the molecules move randomly in any direction and no special coordinate axes are singled out. When the liquid is cooled below a critical (freezing) temperature, a crystal is formed along one particular set of axes. The rotational symmetry between all possible orientations is spontaneously broken in favor of one particular direction.
Symmetry breaking had a powerful explanatory role in developing the Standard Model, where it led to the unification of the fundamental forces at very high energies and to accurate predictions of the existence and properties of new particles, such as the W and Z bosons. In a manner somewhat analogous to the freezing of water, these massive particles are seen to arise as a consequence of symmetry breaking, through the splitting of the unified “electroweak” interaction into the electromagnetic and the weak nuclear forces, as the available energy drops below a critical value.

Individual contributions – Part I

Let us now survey the contents. Following the Introduction the book is divided into four parts. Part I is entitled Continuous Symmetries. Well-chosen texts from Hermann Weyl and Eugene Wigner precede seven articles of unequal length and scope.

- The first, “On continuous symmetries and the foundations of modern physics” by Christopher Martin, covers the entire range indicated by the title and serves as an insightful and competent introduction, not only to the themes of the first section, but also to recurrent subjects throughout the book.
- T. A. Ryckman’s “The philosophical roots of the gauge principle: Weyl and transcendental phenomenological idealism” is the book’s only contribution on the history of symmetry principles in relation to one particular school of philosophy. This interesting piece explores the interrelations between Weyl’s thought and Husserlian phenomenology. It continues and, in part, summarizes research by the author on Weyl and Oskar Becker, in collaboration with Paolo Mancuso, in previous and forthcoming articles.
- The work by Brading and Harvey R. Brown, “Symmetries and Noether’s theorems,” examines the significance of Noether’s seminal paper of 1918 on the connections between the symmetries in Lagrangian dynamics and conservation principles. Besides their inherent interest, these
mathematical results are part of the background required for a full understanding of several of the subsequent articles.

- “General covariance, gauge theories, and the Kretschmann objection” by John D. Norton deals with the role of general covariance in general relativity theory -- an issue related to the notorious “hole argument,” a conceptual obstacle in Einstein’s path to the general theory. This is abundantly discussed elsewhere in numerous papers by this author and several others, including other contributors to this volume, most prominently by John Earman.

- Michael Redhead’s “The interpretation of gauge symmetry” is reprinted here from a 2002 collection of papers on ontological issues of field theories. This is a very insightful examination of the physical and ontological significance of the introduction of “surplus structure,” “ghost fields,” and “unphysical” degrees of freedom in gauge invariant theories at the most fundamental level of current theoretical physics.

- Earman’s “Tracking down gauge: an ode to the constrained Hamiltonian formalism” offers arguments for focusing on theories based on the classical Lagrangian/Hamiltonian approach instead of the most mathematically sophisticated ideas involved in Yang-Mill theories and fibre-bundle formalisms.

- David Wallace’s “Time–dependent symmetries: the link between gauge symmetries and indeterminism” deals with an important difference between local and global symmetries. In gauge theories the requirement of local invariance appears to imply a failure of determinism and the mechanisms by which determinism is restored vary with different theories and types of symmetry. Through his examination of these matters Wallace casts light on the different roles played by symmetry in contemporary physics.

- The tenth and last contribution to Part I is “A fourth way to the Aharonov-Bohm effect” by Antigone Nounou. The mysterious “A-B effect” predicted by Aharonov and Bohm in 1959, and subsequently confirmed by many
experiments, has attracted the attention of philosophers for several reasons. It involves a situation wherein the interference pattern produced by a beam of charged particles can be altered by the action of a constant magnetic field located in a region from which the particles are physically excluded. It appears to confer physical reality onto the gauge potential associated with the field, an entity previously regarded as a mere calculational artifact. The A-B effect is brought to bear in many discussions of nonseparability and holism in quantum physics. Nounou’s contribution is quite technical and uses notions of modern topology to find an explanation for the effect through its relation to the structure of the vacuum state.

Individual contributions – Part II
This part is entitled “Discrete symmetries” and the individual contributions are again preceded by some classic texts: a Leibniz fragment from a letter to Clarke, a short piece by Kant on absolute space, and an extract from a vintage article by Max Black on “The identity of indiscernibles.”

• The first contribution, “Understanding permutation symmetry” by Steven French and Dean Rickles, tackles some long-debated problems concerning the individuality or lack of individuality of quantum particles, in the light of permutation invariance and the principle of the identity of indiscernibles. The discussion advocates a structuralist perspective in which quantum entities are reconceptualized as nodes within a relational network held together by invariance principles. At present many philosophers of science share this perspective, akin to Ladyman’s “structural realism,” with varying qualifications or reservations.

• The following article, “Quarticles and the identity of indiscernibles” by Nick Hugget, elaborates critically on some difficulties found in the preceding one.

• “Handedness, parity violation, and the reality of space” addresses the perennial debates between relationalists and substantivalists on the
nature of space. Oliver Pooley offers here an illuminating historical tour of the problem, from the Leibniz-Claire correspondence, through Kant's famous essay on incongruent counterparts (“enantyomorphs”), down to the impact of the violation of parity symmetry on elementary particle physics.

- Pooley’s article is criticized next in Nick Hugget’s second contribution to the volume, “Mirror symmetry: what is it for a relational space to be orientable?” Hugget takes issue with Pooley’s definition of enantyomorphic parts and with his relationalist view — not in order to embrace a substantivalist stance, but to offer an alternative relationalist account.

Individual contributions – Part III


Within the confines of this review it is impossible to give even a brief account of the issues covered in these deep and closely argued discussions, where the concept of spontaneous symmetry breaking takes central stage.

The first article offers a brief historical introduction by one of the main participants in the initial application of this notion to relativistic quantum field theory by analogy with its role in superconductivity phenomena. Castellani gives a clear account of different forms of symmetry breaking and a review of
the interconnected physical and philosophical issues arising from the role of spontaneous symmetry breaking in causal explanations.

Earman pursues further this examination by exploring the transition from classical physics to quantum field theory and the role of symmetry breaking in the Higgs mechanism, a central feature of the Standard Model.

Morrison’s paper continues these discussions by focusing on the ontological significance of intricate symmetry considerations in establishing the correspondence of theoretical models with physical reality, and the difficulties encountered in conferring a realistic status to our current picture of the structure of the vacuum.

**Individual contributions – Part IV**

Well-chosen fragments of classic texts also precede this last section, “General Interpretative Issues.” They concern the role of invariance in physical theory and are taken from Wigner’s *Symmetry and Reflections*. There are four final contributions: “Symmetry as a guide to superfluous theoretical structure” by Jennan Ismael and Bas Van Fraassen, “Notes on Symmetries” by Gordon Belot, “Symmetry, objectivity and design” by Peter Kosso and “Symmetry and equivalence” by Castellani. These essays focus on some general and pervasive philosophical questions that crop up here and there throughout the entire collection, enmeshed in the details of particular issues.

Among them we reencounter the introduction of superfluous structure, the identity of indiscernibles, symmetries of equations versus symmetries of solutions, effective field theories and scale considerations, and the role of symmetry in the history of physics. Once again, space considerations preclude further discussion here of these thoughtful and thought-provoking essays.
In Conclusion

This work deserves to be in the libraries of all institutions of higher learning that offer graduate level courses in physics or philosophy, and on the shelves of all researchers on the philosophy of physics. Careful and thoughtful editing has resulted in an attractive format, free of the typographical errors and other infelicities that currently so often mar books of this type. Good bibliographies and a detailed and competent index further extend its usefulness.

In anthologies of this nature some unevenness and omissions are almost unavoidable. Since most of the contributors share a common focus and have interacted and debated for years, it is natural for them to ignore other approaches to symmetry in physics (one can think of the work J. Anandan or J. Rosen, for instance). Although most of the contributions are meant to be self-contained, technical background taken for granted in one of the essays is sometimes explained (without cross-reference) in another. It seems it would be possible, at the expense of some repetition, to collect all of these explanatory materials in an introductory section or an appendix. This reviewer would like to see such improvement in the next edition of this commendable collection.

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